# Sensitivity analysis of MAELIA to hydrological parameters

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18th August 2014

The aim of this SA is to check if the sensitivity of the model with snow parameter fixed (by those from SWAT calibration, see work of Youen Grusson).

## The method

The screening method initially developed by (Morris, 1991) and modified by (Campolongo et al., 2007), known as elementary effects method, allows identifying the important factors of a model, including those involved in interactions. It is based on a "One-factor-At-a-Time" (OAT) design of experiments, and is generally used when the number of model parameters is large enough to require computationally expensive simulations. For each parameter, two sensitivity estimates are obtained, both based on the calculation of incremental ratios at various points in the input space of parameters. The incremental ratio is the ratio between the variation of the model output in two different points of the input space (where only one parameter is varied at a time) and the amplitude of the variation of the parameter itself. The Morris method calculates elementary effects ($R_i$) due to each input factor using the following equation:

$$R_i(x_1. \dots .x_n.\Delta) = \frac{y(x_1. \dots .x_{i-1}.x_i + \Delta.x_{i+1}. \dots .x_n) - y(x_1. \dots .x_{i-1}.x_i.x_{i+1}. \dots .x_n)}{\Delta}$$

where $y(X)$ is the output. $X = (x_1. x_2. \dots. x_n)$ is the n-dimensional vector of factors being studied. $\Delta$ is the elementary increment of the OAT.

The method samples values of $X$ from the parameter space to calculate mean ($\mu$, assessing the overall influence of the factor on $y(X)$) and standard deviation ($\sigma$, estimating the totality of the higher order effects, i.e. nonlinearity or interactions with other factors) of all the $R_i$ obtained for each factor. In our case, the exploration of the parameter space was improved by the used of a Latin Hypercube Sampling (LHS), as already illustrated, for example, in (Francos et al., 2003; Van Griensven et al., 2002). Around each point of the LHS of dimension $t$, an OAT is achieved, so the total number of model evaluations needed is $t(n+1)$. In order to improve a bit more the quality of the design we used an LSH, which maximised the 'maximin' criterion (Johnson et al., 1990), i.e. the one with the highest minimum distance of two points of the design. The first sensitivity estimate ($\mu$) is obtained by computing a number of incremental ratios at different points of the input space, and then taking the average of their values. Whereas, the second measure ($\sigma$) is the standard deviation of their values, and is useful to detect parameters either interacting with other parameters, or the effect of which is non-linear (Saltelli et al., 2004). A large measure of central tendency $\mu$ indicates an input with an important overall influence on the output (total effect), while a large $\sigma$ indicates either a parameter with non-linear effect on the output, or a parameter involved in interaction with other parameters (higher than one-order effects). The most relevant parameters are those located in the top right area of the $\sigma$ (spread) versus $\mu$ (strength) plot, where both sensitivity measures are high.

## Our implementation

In our case, the sensitivity analysis was performed over 22 parameters (Table 1), all belongings to the hydrologic sub-model, but not of the snow equations.

To keep the consistency with further sensitivity analysis, the chosen size of the LHS is twelve. The elementary increment of the OAT corresponds to a shift of 1/12 probability over his Uniform distribution. Our number of simulations (288, for 12 local OAT of 23 simulations) is in accordance with literature which suggest at least five OAT for robustness (Confalonieri et al., 2010). As we do not have enough information to chose a distribution, we decided to use a Uniform one as in SWAT literature (e.g. Cibin et al., 2010; Moreau et al., 2013; Muleta and Nicklow, 2005) where authors also recognize that they do not have enough information to determine a distribution curve. However, the use of Uniform distribution is highly common when the main objective is to understand model behaviour (Monod et al., 2006).

Simulations were done over ten years (2000-2009), but the two first years were considered as spin-up period and then ignored. Simulations were simply distributed on a local computer (Quad-Core Intel Xeon: 8 threads with 32 Go RAM), then indices (i.e. $\mu$* which is mean of absolute elementary effects $R_i$ and $\sigma$) were calculated thanks to the 'sensitivity' R package (http://rss.acs.unt.edu/Rdoc/library/sensitivity/html/sensitivity-package.html).

Sensitivity of 11 types of outputs (Table 2) was calculated, by considering scaled parameters (i.e. a [0; 1] parameter range values). The sensitivity was considered either over the full year or during low water period (1st of May-30th September). For each output, sensitivity of the average or the standard deviation, were studied, excepted for dates were the coefficient of variation was used as a proxy of uniformity measure. The hydrologic sensitivity was also check over spatial pattern (e.g. upstream and downstream area)

**Table 1 MAELIA parameter description**

| Parameter | Min value | Max value | Unit | Additional information |
|---|---|---|---|---|
| surlag | 0,5 | 10 | day | Surface runoff lag time |
| nch | 0.01 | 0.1 | - | Manning coefficient value for tributary channels |
| ESCO | 0.01 | 1 | - | Soil evaporation compensation factor |
| EPCO | 0 | 1 | - | Plant uptake compensation factor |
| $\delta_{gw}$ | 0 | 50 | day | Groundwater delay |
| $\beta_{rev}$ | 0.001 | 0.2 | - | Groundwater *revap* coefficient (0.1 for calibration) |
| $\beta_{deep}$ | 0 | 1 | - | Coefficient for percolation to deep aquifer |
| $\alpha_{gw}$ | 0 | 0.3 | - | Baseflow recession constant |

| | | | | |
|---|---|---|---|---|
| $aq_{shthr.rev}$ | 0 | 3000 | mm | Threshold water level in shallow aquifer for revap |
| $aq_{shthr.q}$ | 0 | 1500 | mm | Threshold water level in shallow aquifer for baseflow |
| $CN_2$ | -10 % | +10 % | - | SCS Curve number (one value per landuse: GRASS: 70; FRST: 70; URBN: 65; WATR: 92) |
| $n_{terrain}$ | 0.01 | 0.3 | - | Manning coefficient value for overland flow. |
| msk1 | 0.5 | 1 | - | Muskingum coefficient |
| msk2 | 0 | 0.5 | - | msk2 = 1 – msk1 |
| RCHST | 0 | 10 000 | $m^3$ | Initial water volume of channels. |
| mskX | 0 | 0.5 | - | Muskingum wedge factor |
| LAI | 1.5 | 3 | $m^2/m^2$ | Leaf Area Index for grasslands. |
| SWinit | 0 | 1 | - | Initial soil moisture (fraction of field capacity) |
| tlaps | -8.0 | -4.0 | K/km | Temperature change with altitude |
| plaps | 200 | 800 | mm/km/y | Precipitations change with altitude |
| SHALLST | 0 | 2000 | mm | Initialisation of shallow aquifers. |

**Table 2 Outputs considered in sensitivity analysis**

| Output name | Unit | Description | Full year | Low-flow | Comments |
|---|---|---|---|---|---|
| ET | mm | Real Evapotranspiration | X | X | Sum and standard deviation, weighted by surfaces were considered |
| SwFin | mm | Soil water content | X | X | |
| Perc | mm | Percolation | X | X | |
| eauEntreeAquiferes | mm | Water input in aquifers | X | X | |
| eauAquifereProfond | mm | Deep water aquifers content | X | X | |
| Recap | mm | Capillary rise water amount | X | X | |
| eauStockeeAquiferePeuProfond | mm | Shallow water aquifers content | X | X | |
| ruissellementDeSurfaceHRU | mm | Runoff over an HRU | X | X | |
| ecoulementLateral | mm | Lateral flow | X | X | |
| ecoulementEauSouterraine | mm | Deep water flow | X | X | |
| Water flows | $m^3/s$ | Water flow over 22 sites (which can be used for calibration) | X | X | Sum and standard deviation |

# Results

## Water flow at measure stations

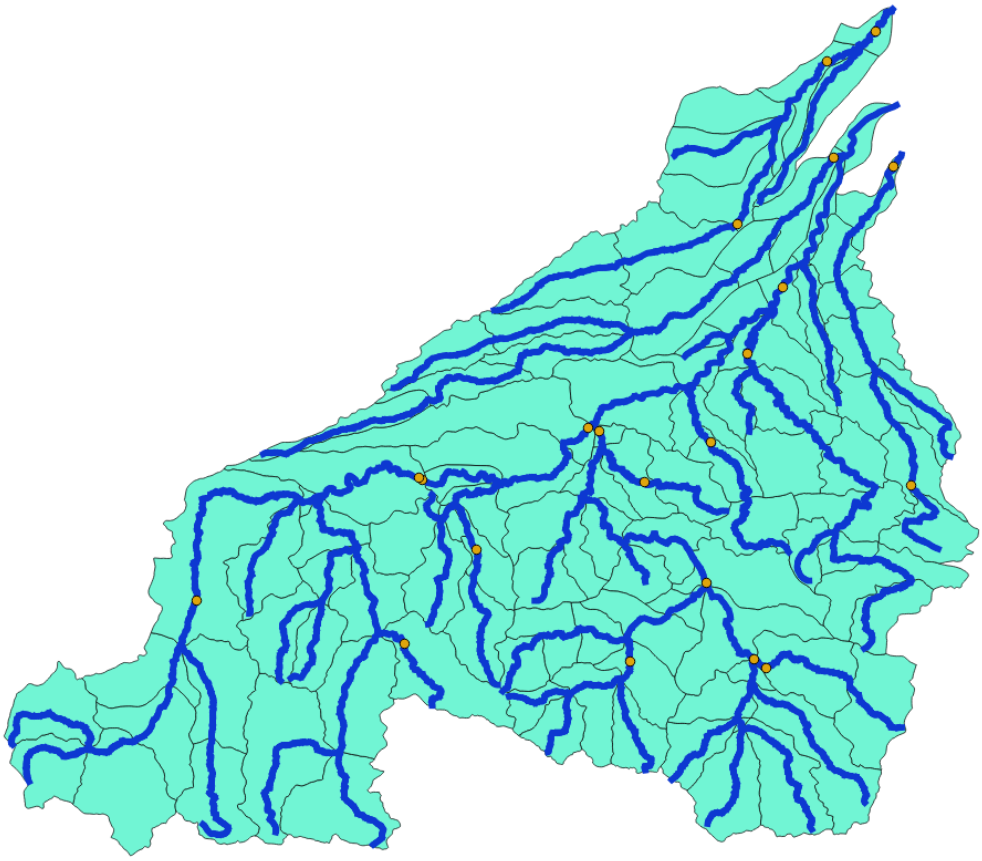

**Figure 1. Water flow sites considered in the sensitivity analysis.**

Considering water flow sensitivity at the different sites (Figure 1), we can find a common set of parameters that seems to drive average values and their dynamics:

- The "*surlag*" parameter, as it drives runoff equations is influent over every site. It is the most influent parameter on the dynamic. It also shows to have a high level of non-linearity or interaction with other parameters.
- The ($\beta_{deep}$) coefficient for percolation to deep aquifer and the to $aq_{shthr.rev}$, threshold of activation of revap equation, are the most influent parameter on the average value of the water flow. Their influence is linked to thier ability to influence the deep aquifers volume. They also shows a high level of non-linearity or interaction with other parameters.
- The curve number "*$CN_2$"* parameter which influences runoff of grassland (*$CN_2$_GRASS*) and forest (*$CN_2$_FRST*) surfaces is mainly influent on the dynamic. The first one is only influent in middle and downstream region (which is due to the fact that upstream region have very few agricultural surfaces), whereas this parameter for forest is highly influent on upstream and middle region, and do not appears as an influent parameters in the downstream region excepted at the outlet site.
- The ground water delay ($\delta_{gw}$) parameter is never the most influent however it is influent on every site. It is influent on the low water period, whereas on annual scale it is not.

- Therefore the exclusion of the two first years, the initial value of shallow aquifers is still influent on the average value. However an easy way to avoid the problem for calibration could be to initiate aquifers with equilibrium values.
- The Manning coefficient for the overland flow is influent on dynamic in most case.

Interestingly, some parameters are only influent on some sites:

- *"LAI"* parameter appears to be influent in middle and downstream region.
- The "*tlaps*" and "*plaps"* parameter (which corrects the climate due to a difference of altitude with the climate grid) had shown a significant influence on upstream region (where in fact the differences to the altitude of reference are higher).

Based on sensitivity results of flows at site levels, we could build a complete list of parameters that are considered influent enough (e.g. at least 10% of the maximum $\mu*$ and $\sigma$). We would get the following list (Table 3). Interestingly, there are very few differences on the order of influent parameters whether we consider water flow on the whole year or only during the "low water period". Only the groundwater delay ("$\delta_{gw}$") seems to be more influent under low water period.

**Table 3. List of parameters considered as influent on water flows**

| | **Frequency of occurrences below the 10% threshold** | | | | | |
|---|---|---|---|---|---|---|
| **Parameter** | All sites, during low water period | All sites, over whole year | On the 3 calibration sites, low water period | Roquefort, low water period | Valentine, low water period | Portet, low water period |
| $aq_{shthr.rev}$ | 93.2% | 93.2% | 100.0% | 100.0% | 100.0% | 100.0% |
| CN2_FRST | 72.7% | 50.0% | 100.0% | 100.0% | 100.0% | 100.0% |
| $\beta_{deep}$ | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% | 100.0% |
| $\delta_{gw}$ | 93.2% | 47.7% | 100.0% | 100.0% | 100.0% | 100.0% |
| SHALLST | 79.5% | 77.3% | 83.3% | 50.0% | 100.0% | 100.0% |
| $n_{terrain}$ | 47.7% | 47.7% | 50.0% | 50.0% | 50.0% | 50.0% |
| surlag | 47.7% | 47.7% | 50.0% | 50.0% | 50.0% | 50.0% |
| $\beta_{rev}$ | 56.8% | 40.9% | 50.0% | 50.0% | 50.0% | 50.0% |
| $aq_{shthr.q}$ | 34.1% | 11.4% | 33.3% | - | - | 50.0% |
| CN2_GRASS | 40.9% | 38.6% | 16.7% | - | - | 50.0% |
| nch | 6.8% | 9.1% | 16.7% | - | - | 50.0% |
| tlaps | 13.6% | 6.8% | 16.7% | 50.0% | - | - |
| LAI | 31.8% | 22.7% | - | - | - | - |
| plaps | 9.1% | 4.5% | - | - | - | - |
| $msk_1$ et $msk_2$ | - | 2.3% | - | - | - | - |

## Water processes and variables

As expected, parameter influence is partly changed whether we consider full year or only the low flow period, so we will mainly focus on low flow period.

As expected the initial water amount (*SHALLST*) is influent on shallow aquifer. The $\beta_{deep}$, $\beta_{rev}$ and $aq_{shthr.q}$ parameter are influent on both shallow and deep aquifers. Shallow aquifers are also sensitive to $aq_{shthr.rev}$. The ground water delay is also influent on deep aquifers dynamic.

Water aquifer input dynamic is mainly to the groundwater delay ($\delta_{gw}$), whereas its average value is mainly linked to $aq_{shthr.q}$ parameter, and $\beta_{rev}$.

Deep-water flow is, as expected, highly sensitive to $\beta_{deep}$, $\beta_{rev}$ , $aq_{shthr.rev}$ and $aq_{shthr.q}$.

The evapotranspiration sensitivity is low and is mainly influenced by the *LAI* parameter. Surprisingly, we could note a low sensitivity to the EPCO and ESCO factors.

Lateral flow shows is sensitive to the curve number (*$CN_2$_FRST*) of forest and to parameters of influence the aquifers ($\beta_{deep}$, $\beta_{rev}$ , $aq_{shthr.rev}$ and $aq_{shthr.q}$).

Percolation process showed to be sensitive to $\beta_{rev}$, *LAI*, *SHALLST*, *$CN_2$_FRST*, and $aq_{shthr.q}$.

The capillary rise is one of the most sensitive process (i.e. which covers a wide relative variation range). Its most influent parameters are $aq_{shthr.q}$ $\beta_{rev}$ ,$aq_{shthr.rev}$ and *SHALLST*.

Runoff sensitivity is dominated by the *surlag*, the curve number factors (*$CN_2$_FRST* and *$CN_2$_GRASS*), the dynamic is also well influenced by $n_{terrain}$.

The soil water amount sensitivity is firstly dominated by $\beta_{rev}$. Some other parameters such as $aq_{shthr.,q}$ ,$aq_{shthr.rev}$ ,*LAI* and *SHALLST* show a high non-linearity or a high level of interaction with other parameters on this process.

One can ask whether the sensitivity would significantly different whether we consider the whole area or the up or downstream region. To check this pattern, we also performed the analysis on two sub-areas (Figure 2).

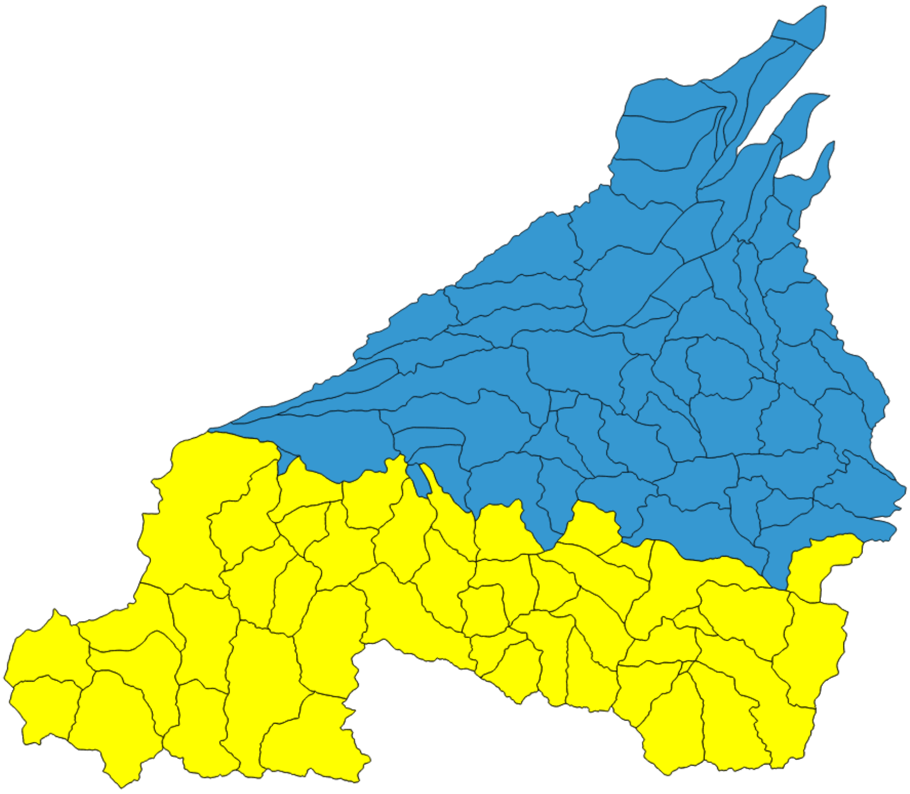

**Figure 2. Split of the area on upstrean and downstream region**

As expected some process or output show the same sensitivity on both area (deep and shallow aquifers water content, deep-water flow, capillary rise), whereas for others (water aquifer input, lateral flow, percolation, runoff) we can distinguish two patterns:

- the curve number for forest (*$CN_2$_FRST*) is highly influent in the upper part
- the curve number for grassland (CN2_GRASS) and their leaf area index (LAI) for the downstream part

Interestingly the soil water amount (simulated by SWAT) sensitivity is highly different between up- and downstream region. It is three times more sensitive in downstream region than in the up-stream region.

## References

Campolongo, F., Cariboni, J., Saltelli, A., 2007. An effective screening design for sensitivity analysis of large models. Environ. Model. Softw. 22, 1509–1518.

Cibin, R., Sudheer, K.P., Chaubey, I., 2010. Sensitivity and identifiability of stream flow generation parameters of the SWAT model. Hydrol. Process. 24, 1133–1148.

Confalonieri, R., Bellocchi, G., Bregaglio, S., Donatelli, M., Acutis, M., 2010. Comparison of sensitivity analysis techniques: a case study with the rice model WARM. Ecol. Model. 221, 1897–1906.

Francos, A., Elorza, F.J., Bouraoui, F., Bidoglio, G., Galbiati, L., 2003. Sensitivity analysis of distributed environmental simulation models: understanding the model behaviour in hydrological studies at the catchment scale. Reliab. Eng. Syst. Saf. 79, 205–218.

Guse, B., Reusser, D.E., Fohrer, N., 2013. How to improve the representation of hydrological processes in SWAT for a lowland catchment - temporal analysis of parameter sensitivity and model performance. Hydrol. Process.

Johnson, M.E., Moore, L.M., Ylvisaker, D., 1990. Minimax and maximin distance designs. J. Stat. Plan. Inference 26, 131–148.

Monod, H., Naud, C., Makowski, D., 2006. Uncertainty and sensitivity analysis for crop models, in: Wallack, D., Makowski, D., Jones, J.W. (Eds.), Working with Dynamic Crop Models. Elsevier, Amsterdam; Boston, pp. 55–100.

Moreau, P., Viaud, V., Parnaudeau, V., Salmon-Monviola, J., Durand, P., 2013. An approach for global sensitivity analysis of a complex environmental model to spatial inputs and parameters: A case study of an agro-hydrological model. Environ. Model. Softw. 47, 74–87.

Morris, M.D., 1991. Factorial Sampling Plans for Preliminary Computational Experiments. Technometrics 33, 161–174.

Muleta, M.K., Nicklow, J.W., 2005. Sensitivity and uncertainty analysis coupled with automatic calibration for a distributed watershed model. J. Hydrol. 306, 127–145.

Saltelli, A., Tarantola, S., Campolongo, F., Ratto, M., 2004. Sensitivity analysis in practice: a guide to assessing scientific models. John Wiley & Sons, Hoboken, NJ.

Van Griensven, A., Francos, A., Bauwens, W., 2002. Sensitivity analysis and auto-calibration of an integral dynamic model for river water quality. Water Sci. Technol. 45, 325–332.

Van Griensven, A., Meixner, T., Grunwald, S., Bishop, T., Diluzio, M., Srinivasan, R., 2006. A global sensitivity analysis tool for the parameters of multi-variable catchment models. J. Hydrol. 324, 10–23.